

# Localization of massive and massless fermion on two field brane

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## Abstract

In this paper we study fermion localization and resonances on a special type of braneworld model supporting brane splitting. In such models one can construct multi-wall branes which cause considerable simplification in field equations. We use a polynomial superpotential to construct this brane. The suitable Yukawa coupling between the background scalar field and localized fermion is determined. The massive fermion resonance spectrum is obtained. The number of resonances is increased for higher values of Yukawa coupling.

**PACS:**

**Key Words:** Fermion localization, Resonances, Braneworld model, Yukawa coupling

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# 1 Introduction

Braneworld scenarios have attracted considerable attention in the literature during the last two decades because these models can address some important issues in theoretical physics problems such as hierarchy [1, 2, 3] and cosmological constant problem [4, 5]. The branes in Randall and Sundrum (RS) models are fixed in some points along extra dimension and have a  $\delta$  function form [1, 6]. This brane-world model is very ideal and its formation has no dynamical mechanism. But for realistic models thickness of brane should be considered. By now several thick brane construction mechanisms have been developed such as thick branes generated from pure gravity [7]-[10], fermion self interaction branes [11, 12] and thick brane scenarios with the gravity coupled to five-dimensional scalar fields [13]-[20]. In the last scenario, the scalar field configuration is usually a kink. It is found that a single kink becomes unstable when it moves in a discrete lattice with a large velocity while multi-kink solutions remain stable [21]. This phenomenon is associated with interaction between kink and radiation, and the resonances were observed experimentally [22, 23]. Furthermore, in cosmology we encounter models in which our universe is result of continuous collision of branes and nucleation and therefore splitting of branes is a fundamental scenario in these models [24]-[27].

Therefore, the universal aspects of thick brane splitting in warped bulk is important. such branes are constructed from a complex  $\phi^4$  scalar field potential [28], or from a real  $\phi^6$  scalar field potential [29]. These branes can be constructed from deformation of  $\phi^4$  scalar field potential as well.[30]-[32].

Recently, Dutra and co-workers proposed a new model of thick brane in which multi-brane scenario arises from scalar field models generating usual kink solutions [33]. It suggests a special type of brane splitting. In this method superpotential function and warp factor will decompose in a special form and field equations will be simplified significantly. In this work we deal with this thick brane model which arises from polynomial superpotential.

The localization of spin 1/2 fermions on the brane is very interesting and important. Usually, in five dimension fermion do not have a normalizable zero mode without scalar-fermion coupling [34]-[42]. In five dimension, with a Yukawa scalar-fermion coupling there may exist a massless bound state and a continuous gapless spectrum of massive Kaluza-Klein (KK) states [38, 43], while in some of other brane models, there exist some discrete KK state and a continuous gapless mass spectrum starting from positive  $m^2$  [44, 45].

This paper is organized as follows. In the next section, we present the brane model that is constructed in Ref. [33] In section 3. we investigate localization of the zero mode of the fermion field on the brane which is derived from polynomial potential. In section 4 we study localization of massive fermionic mode. Finally, in the last section we present our conclusions.

## 2 The model

we consider the following action in which two scalar fields coupled to gravity in 5 dimensions

$$S = \int d^4x dr \sqrt{-g} \left[ \frac{1}{4} R - \frac{1}{2} \partial_M \phi_1 \partial^M \phi_1 - \frac{1}{2} \partial_M \phi_2 \partial^M \phi_2 - V(\phi_1, \phi_2) \right] \quad (1)$$

where  $g = \det(g_{MN})$ ,  $M, N = 0, 1, 2, 3, 4$ . The coordinates in the brane is represented by  $x^\mu$  ( $\mu = 0, 1, 2, 3$ ) while the coordinate in the bulk is shown by  $x^4 = r$ . the line element is written as

$$ds^2 = g_{MN}dx^M dx^N = e^{2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu + dr^2, \quad (2)$$

where  $\eta_{\mu\nu}$  is usual Minkowski metric with  $\text{diag}(-, +, +, +)$  and  $e^{2A(r)}$  is called the warp factor. For this braneworld scenario, the equations of motion is obtained as

$$\phi_i'' + 4A'\phi_i' = \frac{dV}{d\phi_i}. \quad (i = 1, 2) \quad (3)$$

$$A'' + \frac{2}{3}(\phi_1'^2 + \phi_2'^2) = 0. \quad (4)$$

$$A'^2 + \frac{1}{3}V(\phi_1, \phi_2) = \frac{1}{6}(\phi_1'^2 + \phi_2'^2). \quad (5)$$

the potential  $V(\phi_1, \phi_2)$  can be written in terms of a superpotential  $W(\phi_1, \phi_2)$  as

$$V(\phi_1, \phi_2) = \frac{1}{2} \sum_{i=1}^2 \left( \frac{\partial W}{\partial \phi_i} \right)^2 - \frac{4}{3}W^2 \quad (6)$$

Therefore, equations of motion can be reduced to the following first order equations

$$\frac{d\phi_i}{dr} = \frac{\partial W}{\partial \phi_i}, \quad \frac{dA}{dr} = -\frac{2}{3}W. \quad (i = 1, 2) \quad (7)$$

But

$$W(\phi_1, \phi_2) = W_1(\phi_1) + W_2(\phi_2), \quad A(r) = A_1(r) + A_2(r) \quad (8)$$

the first order equations in (7) are converted to

$$\frac{d\phi_i}{dr} = \frac{\partial W_i(\phi_i)}{\partial \phi_i}, \quad \frac{dA_i}{dr} = -\frac{2}{3}W_i(\phi_i). \quad (i = 1, 2) \quad (9)$$

for polynomial superpotential

$$W_i(\phi_i) = \lambda_i(\phi_i - \frac{\phi_i^3}{3}) \quad (i = 1, 2) \quad (10)$$

$\phi_i$  and  $A_i$  is given by:

$$\phi_i = \tanh[\lambda_i(r - r_i)] \quad (i = 1, 2) \quad (11)$$

$$A(r) = \frac{1}{9} \sum_{i=1}^2 \{ \tanh^2[\lambda_i(\tilde{r} - r_i)] - \tanh^2[\lambda_i(r - r_i)] \} - \frac{4}{9} \ln \left( \prod_{i=1}^2 \frac{\text{sech}[\lambda_i(\tilde{r} - r_i)]}{\text{sech}[\lambda_i(r - r_i)]} \right) \quad (12)$$

where  $r_i$  is an integration constant, representing the center of the kink and  $\tilde{r}$  is defined as the average value of coordinates of center of the kinks

$$\tilde{r} = \frac{1}{2}(r_1 + r_2) \quad (13)$$

In order to this brane model support brane splitting mechanism, we consider two symmetric kink solutions. Therefore,  $\lambda_1 = \lambda_2 = \lambda$  and  $r_1 = -r_2 = a$ . So  $\tilde{r} = 0$  and we can write equations (11) and (12) as

$$\phi_1 = \tanh[\lambda(r - a)], \quad \phi_2 = \tanh[\lambda(r + a)] \quad (14)$$

$$\begin{aligned}
A(r) &= \frac{1}{9} \{ 2 \tanh^2(\lambda a) - \tanh^2[\lambda(r-a)] - \tanh^2[\lambda(r+a)] \} \\
&- \frac{4}{9} \ln \left( \frac{\cosh[\lambda(r-a)] \cosh[\lambda(r+a)]}{\cosh^2(\lambda a)} \right)
\end{aligned} \tag{15}$$

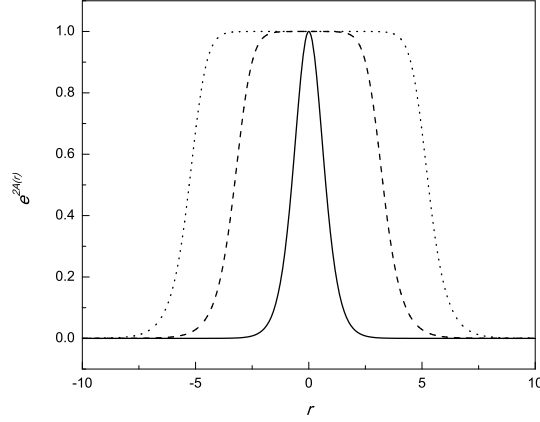


Figure 1: plots of warp factor  $e^{2A(r)}$  for  $a = 0$  (solid line),  $a = 3.0$  (dashed line) and  $a = 5.0$  (dotted line). we put  $\lambda = 1$

Fig. 1 represents warp factor for different values of  $a$ . For  $a = 0$  we have a single brane that warp factor has a sharp peak. For  $a > 0$  a plateau is formed in the very inside the brane where the energy density vanishes. This is attributed to presence a new phase inside the brane. The plateau region inside the brane grows when  $a$  increase.

In the next section, we investigate localization of zero mode of the fermion on the brane.

### 3 Localization of zero mode

Let us consider the action of fermion coupled to gravity and the background scalar fields

$$S_f = \int d^5x \sqrt{-g} [\bar{\Psi} \Gamma^M D_M \Psi - \eta \bar{\Psi} F(\phi_1, \phi_2) \Psi], \tag{16}$$

where  $\eta$  is coupling constant. Here the back- reaction effect of fermion on background scalar field is neglected and scalar field is considered to be unchanged.  $\Gamma_M$  is general gamma matrices,  $\Gamma^M = (e^{-A} \gamma^\mu, e^{-A} \gamma^4)$ .

By following coordinate transformation

$$dz = e^{-A} dr \tag{17}$$

the metric in (2) is changed to conformally flat one.

$$ds^2 = e^{2A} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2) \tag{18}$$

the equation of motion for fermion is derived as

$$[\gamma^\mu \partial_\mu + \gamma^4 (\partial_z + 2\partial_z A) - \eta e^A F(\phi_1, \phi_2)] \Psi = 0. \tag{19}$$

For solving this equation, we separate variables with KK and chiral decomposition

$$\Psi(x, z) = \sum_n \left( \psi_{Ln}(x) f_{Ln}(z) + \psi_{Rn}(x) f_{Rn}(z) \right) \quad (20)$$

The 4D left-handed and right-handed fermions satisfy the Dirac equations

$$\begin{aligned} \gamma^\mu \partial_\mu \psi_{Ln} &= m_n \psi_{Rn} \\ \gamma^\mu \partial_\mu \psi_{Rn} &= m_n \psi_{Ln} \end{aligned} \quad (21)$$

while the KK modes satisfy

$$\{\partial_z + 2\partial_z A + \eta e^A F(\phi_1, \phi_2)\} f_{Ln}(z) = m_n f_{Rn}(z) \quad (22)$$

$$\{\partial_z + 2\partial_z A - \eta e^A F(\phi_1, \phi_2)\} f_{Rn}(z) = -m_n f_{Ln}(z) \quad (23)$$

with the following ortho-normality condition

$$\begin{aligned} \int_{-\infty}^{\infty} dz e^{4A} f_{Ln} f_{Lm} &= \int_{-\infty}^{\infty} dz e^{4A} f_{Rn} f_{Rm} = \delta_{nm}, \\ \int_{-\infty}^{\infty} dz e^{4A} f_{Ln} f_{Rm} &= 0 \end{aligned} \quad (24)$$

and defining  $\tilde{f}_{Ln} = e^{2A} f_{Ln}$  and  $\tilde{f}_{Rn} = e^{2A} f_{Rn}$ , the Schrödinger-like equations are obtained.

$$[-\partial_z^2 + V_L(z)] \tilde{f}_{Ln} = m_n^2 \tilde{f}_{Ln} \quad (25)$$

$$[-\partial_z^2 + V_R(z)] \tilde{f}_{Rn} = m_n^2 \tilde{f}_{Rn} \quad (26)$$

where the effective potentials are given by

$$\begin{aligned} V_L(z) &= \eta^2 e^{2A} F^2(\phi_1, \phi_2) - \eta \partial_z [e^A F(\phi_1, \phi_2)] \\ V_R(z) &= \eta^2 e^{2A} F^2(\phi_1, \phi_2) + \eta \partial_z [e^A F(\phi_1, \phi_2)] \end{aligned} \quad (27)$$

Because of the complexity of  $A(r)$ , we can not use relation (17) to obtain analytical form of  $z(r)$ . Therefore we apply numerical method to get pair of  $(r, z)$ . with realations

$$\partial_z A = e^{A(r)} \partial_r A, \quad \partial_z F = e^{A(r)} \partial_r F \quad (28)$$

By these relations, we can rewrite the potentials as the function of  $r$

$$\begin{aligned} V_L(z(r)) &= \eta e^{2A} [\eta F^2 - \partial_r F - F \partial_r A(r)] \\ V_R(z(r)) &= \eta e^{2A} [\eta F^2 + \partial_r F + F \partial_r A(r)] \end{aligned} \quad (29)$$

With substituting  $m_n = 0$  in (22) and (23), the left-handed and right-handed zero mode of fermion are obtained as:

$$f_{L0} \propto \exp \left[ -\eta \int_0^z dz' e^{A(z')} F(\phi_1, \phi_2) \right] \quad (30)$$

$$f_{R0} \propto \exp \left[ \eta \int_0^z dz' e^{A(z')} F(\phi_1, \phi_2) \right] \quad (31)$$

The normalization condition for localizing zero mode of left-handed fermions on the brane is

$$\int_{-\infty}^{\infty} dz \exp \left( -2\eta \int_0^z dz' e^{-A(z')} F(\phi_1(z'), \phi_2(z')) \right) < \infty \quad (32)$$

in  $r$  coordinate, we have:

$$\int_{-\infty}^{\infty} dr \exp \left( -A(r) - 2\eta \int_0^r dr' F(\phi_1(r'), \phi_2(r')) \right) < \infty \quad (33)$$

For right handed this condition is achieved by replacing  $\eta \rightarrow -\eta$ . For studying localization of zero mode of fermion on the brane, we should determine the suitable form of  $F(\phi_1, \phi_2)$ . In the following subsections we try to determine this function using normalization condition.

### 3.1 $F = \phi_1 \phi_2$

The integrand in (33) can be expressed as

$$\begin{aligned} I_0 &\equiv \exp \left( -A(r) - 2\eta \int_0^r dr' F(\phi_1(r'), \phi_2(r')) \right) \\ &= \exp \left[ -\frac{1}{9} \{ 2 \tanh^2(\lambda a) - \tanh^2[\lambda(r-a)] - \tanh^2[\lambda(r+a)] \} - 2\eta r \right] \\ &\times \left( \frac{\cosh[\lambda(r-a)] \cosh[\lambda(r+a)]}{\cosh^2(\lambda a)} \right)^{\frac{4}{9}} \left( \frac{1 + e^{-2\lambda(x-a)}}{1 + e^{-2\lambda(x+a)}} \right)^{\frac{-2\eta}{\lambda}} \end{aligned} \quad (34)$$

We can decompose normalization condition to two regions as

$$\int_0^{\infty} dr I_0 < 0, \quad \int_{-\infty}^0 dr I_0 < 0 \quad (35)$$

from first integral we have

$$I_0 \rightarrow \exp(8/9 - 2\eta r) \quad \text{when} \quad r \rightarrow +\infty \quad (36)$$

For satisfying normalization condition, we require  $\eta > 4/9$ . For second integral we have

$$I_0 \rightarrow \exp(-8/9 - 2\eta r) \rightarrow \infty \quad \text{when} \quad r \rightarrow -\infty \quad (37)$$

therefore the second integral is divergent. So the left-handed fermion zero mode can not be localized on the brane. In the other hand, with  $\eta \rightarrow -\eta$ , we can see that the zero mode of right handed fermion can not localize too.

### 3.2 $F = \phi_1 - \phi_2$

the integrand in (33) is written as

$$\begin{aligned} I_1 &\equiv \exp \left[ -\frac{1}{9} \{ 2 \tanh^2(\lambda a) - \tanh^2[\lambda(r-a)] - \tanh^2[\lambda(r+a)] \} \right] \\ &\times \left( \frac{\cosh[\lambda(r-a)] \cosh[\lambda(r+a)]}{\cosh^2(\lambda a)} \right)^{\frac{4}{9}} \left( \frac{\cosh[\lambda(r-a)]}{\cosh[\lambda(r+a)]} \right)^{-\frac{2\eta}{\lambda}} \end{aligned} \quad (38)$$

from which we have

$$I_1 \rightarrow \exp(\pm 8r/9) \rightarrow \infty \quad \text{when} \quad r \rightarrow \pm\infty \quad (39)$$

Hence, left-handed zero mode can not be localized on the brane in this case. Moreover, because  $I_1$  is independent of  $\eta$  therefore, we can conclude that right handed fermion can not be localized on the brane.

### 3.3 $F = \phi_1 + \phi_2$

The integrand in (33) is expressed as

$$\begin{aligned} I_2 &\equiv \exp\left[-\frac{1}{9}\{2\tanh^2(\lambda a) - \tanh^2[\lambda(r-a)] - \tanh^2[\lambda(r+a)]\}\right] \\ &\times \left(\frac{\cosh[\lambda(r-a)] \cosh[\lambda(r+a)]}{\cosh^2(\lambda a)}\right)^{\frac{4}{9} - \frac{2\eta}{\lambda}} \end{aligned} \quad (40)$$

therefore, we have:

$$I_2 \rightarrow \exp[\pm(8r/9 - 4\eta r/\lambda)] \quad \text{when} \quad r \rightarrow \pm\infty \quad (41)$$

So the normalization condition for localization of left handed fermion zero mode is

$$\eta > \frac{2\lambda}{9} \quad (42)$$

By changing  $\eta \rightarrow -\eta$  one can find that the zero mode of right handed fermion, however this mode can not be localized on the brane.

The effective potentials for left handed and right handed KK fermion have the form

$$\begin{aligned} V_L &= \eta \exp\left[\frac{2}{9}\{2\tanh^2(\lambda a) - \tanh^2[\lambda(r-a)] - \tanh^2[\lambda(r+a)]\}\right] \\ &\times \left(\frac{\cosh \lambda(r-a) \cosh[\lambda(r+a)]}{\cosh^2(\lambda a)}\right)^{-\frac{8}{9}} \left\{ \eta (\tanh[\lambda(r-a)] + \tanh[\lambda(r+a)])^2 \right. \\ &- \lambda \left( \text{sech}^2[\lambda(r-a)] + \text{sech}^2[\lambda(r+a)] + \frac{2}{3} (\tanh[\lambda(r-a)] + \tanh[\lambda(r+a)]) \right. \\ &\times \{ \tanh[\lambda(r-a)] + \tanh[\lambda(r+a)] \\ &- \left. \frac{\tanh^3[\lambda(r-a)] + \tanh^3[\lambda(r+a)]}{3} \} \} \right\} \end{aligned} \quad (43)$$

$$V_R = V_L|_{\eta \rightarrow -\eta} \quad (44)$$

The Values of potential in  $r$  or  $z = 0$  and  $r$  or  $z \rightarrow \pm\infty$  are

$$V_L(0) = -V_R(0) = -2\eta\lambda \text{sech}^2(\lambda a) \quad (45)$$

$$V_L(\pm\infty) = V_R(\pm\infty) = 0 \quad (46)$$

It can be seen that the asymptotic behaviors of two potentials are the same when  $y \rightarrow \pm\infty$ , but opposite at the origin,  $z = 0$ . This reveals that only one of the massless left and right chiral fermions can be localized on the brane. The shape of effective potentials are shown in Fig. 2. The form of  $V_L(z)$  is volcano type and therefore there is no mass gap between the zero mode and KK excitation modes. On the other hand, the  $V_R(z)$  is always positive at the brane location. We know

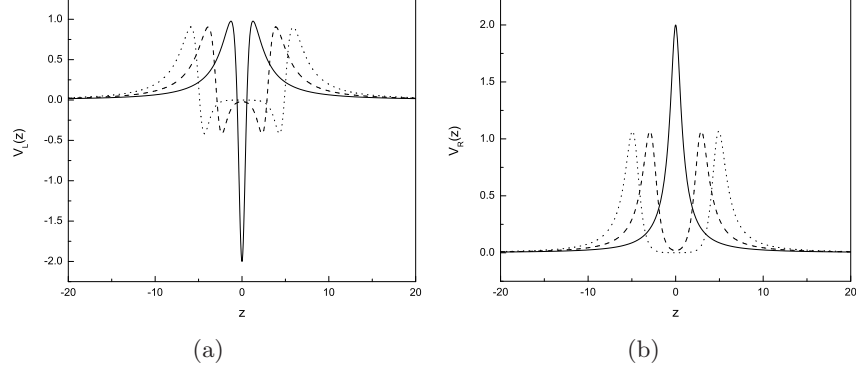


Figure 2: The shapes of potentials: (a)  $V_L(z)$  and (b)  $V_R(z)$ ,  $a = 0$  (solid line),  $a = 3$  (dashed line) and  $a = 5$  (dotted line).

that this type of potential can not trap any bound state of right handed fermion and there is no zero mode of right handed fermion. This is consistent with the previous our knowledge that only one chirality of massless fermion can exist.

The zero mode of left-handed fermion is written as

$$\begin{aligned}
 \tilde{f}_{L0}(z) &\propto \exp\left(-\eta \int_0^z dz' e^{A(z')} [\phi_1(z') + \phi_2(z')]\right) \\
 &= \exp\left(-\eta \int_0^r dr' [\phi_1(r') + \phi_2(r')]\right) \\
 &= \left(\cosh[\lambda(r-a)] \cosh[\lambda(r+a)]\right)^{-\frac{2\eta}{\lambda}}
 \end{aligned} \tag{47}$$

Fig. 3 shows the form of fermion zero modes on the brane. One can see that the width of the function is increased with splitting and the height is decreased.

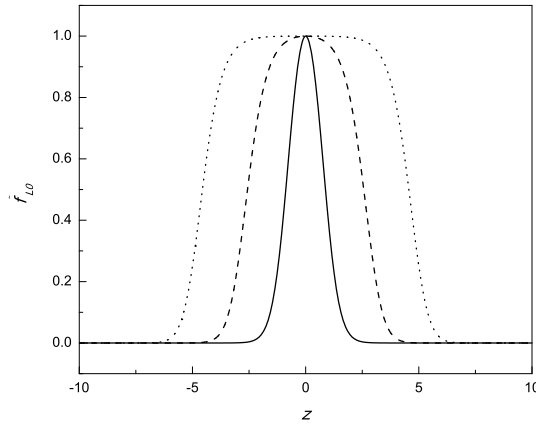


Figure 3: Fermion zero modes localized on the brane :  $a = 1$  (solid line),  $a = 3$  (dashed line) and  $a = 5$  (dotted line).



### 3.4 The case $F = \phi_1 + \beta\phi_2$

The integrand in (33) is expressed as:

$$I_3 \equiv \exp\left[-\frac{1}{9}\{2\tanh^2(\lambda a) - \tanh^2[\lambda(r-a)] - \tanh^2[\lambda(r+a)]\}\right] \\ \times \left(\frac{\cosh[\lambda(r-a)]}{\cosh(\lambda a)}\right)^{\frac{4}{9}-\frac{2\eta}{\lambda}} \left(\frac{\cosh[\lambda(r+a)]}{\cosh(\lambda a)}\right)^{\frac{4}{9}-\beta\frac{2\eta}{\lambda}} \quad (48)$$

Therefore, we have:

$$I_3 \rightarrow \exp\left[\pm(8r/9 - 2(\beta+1)\eta r/\lambda)\right] \quad \text{when} \quad r \rightarrow \pm\infty \quad (49)$$

So the normalization condition becomes :

$$\eta > \frac{4\lambda}{9(\beta+1)} \quad (50)$$

For  $\beta = -1$  the condition (50) can not be satisfied, Hence, left-handed zero mode can not be localized on the brane. for  $\beta = 0$  we have  $\eta > 4\lambda/9$ . This means that coupling of fermion to every sub-brane can localize zero mode on the brane. For the case  $\beta = 1$  the normalization condition is reduced to eq. (42). The zero mode of left-handed fermion is turned out to be

$$\tilde{f}_{L0}(z) \propto \exp\left(-\eta \int_0^r dr' [\phi_1(r') + \beta\phi_2(r')]\right) \\ = \cosh[\lambda(r-a)]^{-\frac{2\eta}{\lambda}} \cosh[\lambda(r+a)]^{-\frac{2\beta\eta}{\lambda}} \quad (51)$$

The explicit forms of the potentials are

$$V_L = \eta \exp\left[\frac{2}{9}\{2\tanh^2(\lambda a) - \tanh^2[\lambda(r-a)] - \tanh^2[\lambda(r+a)]\}\right] \\ \times \left(\frac{\cosh \lambda(r-a) \cosh[\lambda(r+a)]}{\cosh^2(\lambda a)}\right)^{-\frac{8}{9}} \left\{ \eta (\tanh[\lambda(r-a)] + \beta \tanh[\lambda(r+a)])^2 \right. \\ - \lambda \left( \text{sech}^2[\lambda(r-a)] + \beta^2 \text{sech}^2[\lambda(r+a)] + \frac{2}{3} (\tanh[\lambda(r-a)] + \beta \tanh[\lambda(r+a)]) \right. \\ \left. \left. \times \{ \tanh[\lambda(r-a)] + \tanh[\lambda(r+a)] - \frac{\tanh^3[\lambda(r-a)] + \tanh^3[\lambda(r+a)]}{3} \} \right) \right\} \quad (52)$$

$$V_R = V_L|_{\eta \rightarrow -\eta} \quad (53)$$

The asymptotic behaviors of the potentials are

$$V_L(0) = \eta \left\{ \eta(\beta-1)^2 - [\eta(\beta-1)^2 + \lambda(\beta^2+1)] \text{sech}^2(\lambda a) \right\} \quad (54)$$

$$V_R(0) = \eta \left\{ \eta(\beta-1)^2 - [\eta(\beta-1)^2 - \lambda(\beta^2+1)] \text{sech}^2(\lambda a) \right\} \quad (55)$$

$$V_L(\pm\infty) = V_R(\pm\infty) = 0 \quad (56)$$

It is realized that  $V_L(0)$  can be negative or positive, while for  $\eta > 0$  and  $\lambda > 0$ ,  $V_R(0)$  is always greater than  $V_L(0)$ .

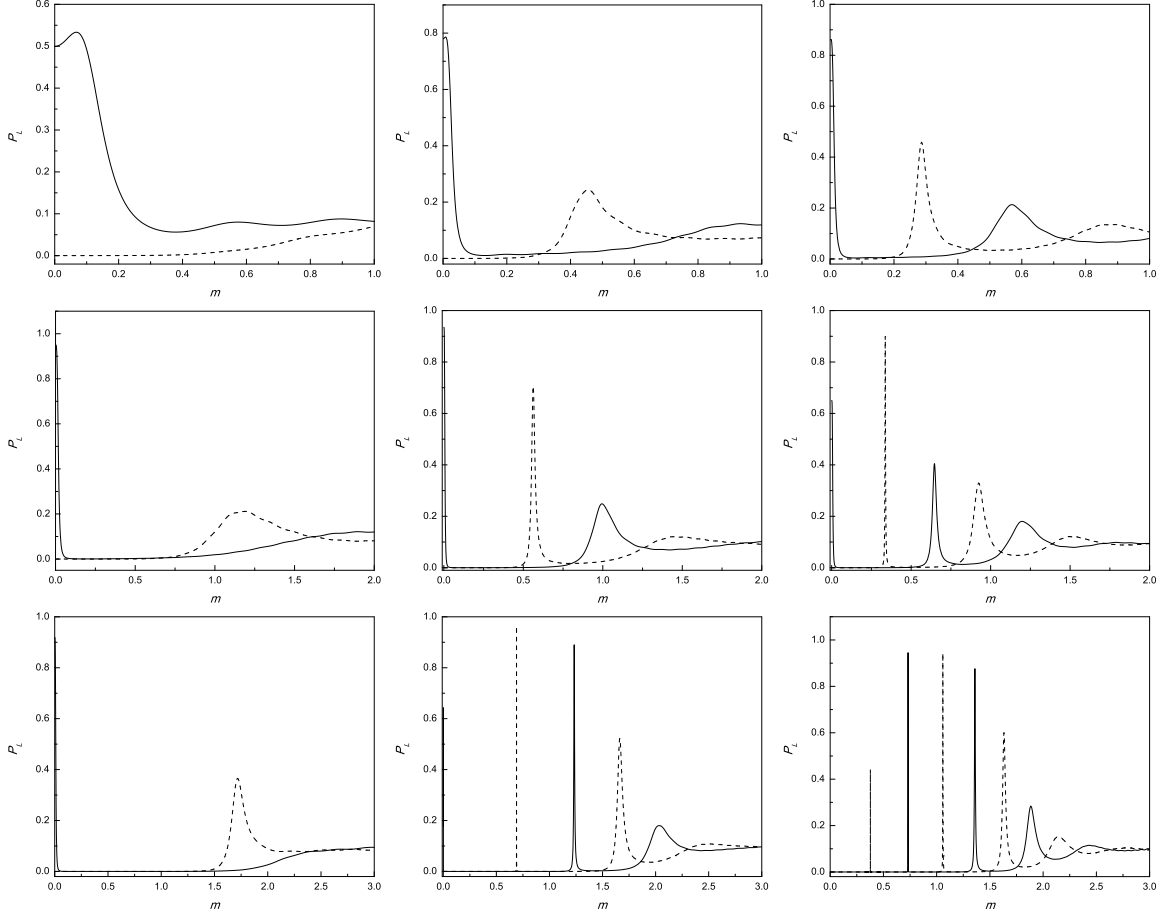


Figure 4: Plots of relative probability for left handed fermion resonances with  $a = 1.0$ (left column),  $a = 3.0$ (middle column) and  $a = 5.0$ (right column). Coupling constants are  $\eta = 0.5$ (first row),  $\eta = 1.0$ (second row) and  $\eta = 2.0$ (third row). Solid lines and dashed lines corresponds to even and odd parity respectively. In all cases  $\lambda = 1.0$ .

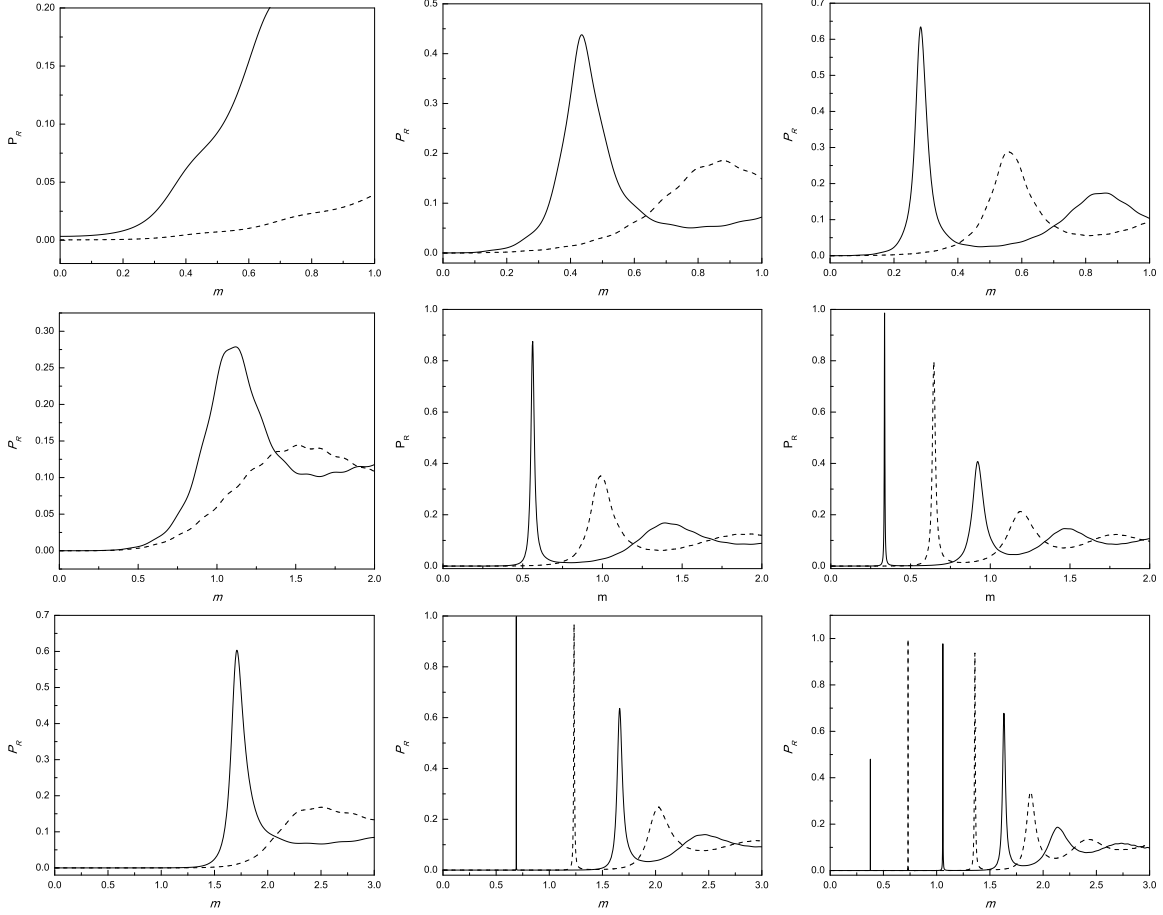


Figure 5: Plots of relative probability for right handed fermion resonances with  $a = 1.0$ (left column),  $a = 3.0$ (middle column) and  $a = 5.0$ (right column). Coupling constants are  $\eta = 0.5$ (first row),  $\eta = 1.0$ (second row) and  $\eta = 2.0$ (third row). Solid lines and dashed lines corresponds to even and odd parity respectively. In all cases  $\lambda = 1.0$ .

## 4 Resonances of massive modes

We can rewrite eqs. (25) and (26) as

$$\begin{aligned} Q^\dagger Q \tilde{f}_{Ln} &= (-\partial_z + \eta F e^A)(\partial_z + \eta F e^A) \tilde{f}_{Ln} = m^2 \tilde{f}_{Ln} \\ Q Q^\dagger \tilde{f}_{Rn} &= (\partial_z + \eta F e^A)(-\partial_z + \eta F e^A) \tilde{f}_{Rn} = m^2 \tilde{f}_{Rn} \end{aligned} \quad (57)$$

From these equation, we can see that the tachyonic modes in spectrum excluded. With converting the equations of motion for fermion to Schrödinger-like equations, we can present a quantum mechanical interpretation for  $\tilde{f}_{Ln}$  and  $\tilde{f}_{Rn}$ . By studying resonant modes we are able to obtain information about the coupling between massive modes and the brane.

In order to derive KK modes from equation (25) and (26) we apply relative probability method [46, 47, 48]. Since the equations (25) and (26) are Schrödinger-like, we can interpret normalized  $|\tilde{f}_{L,R}(z)|^2$  as the propability of finding massive KK modes on the brane. But the massive modes can

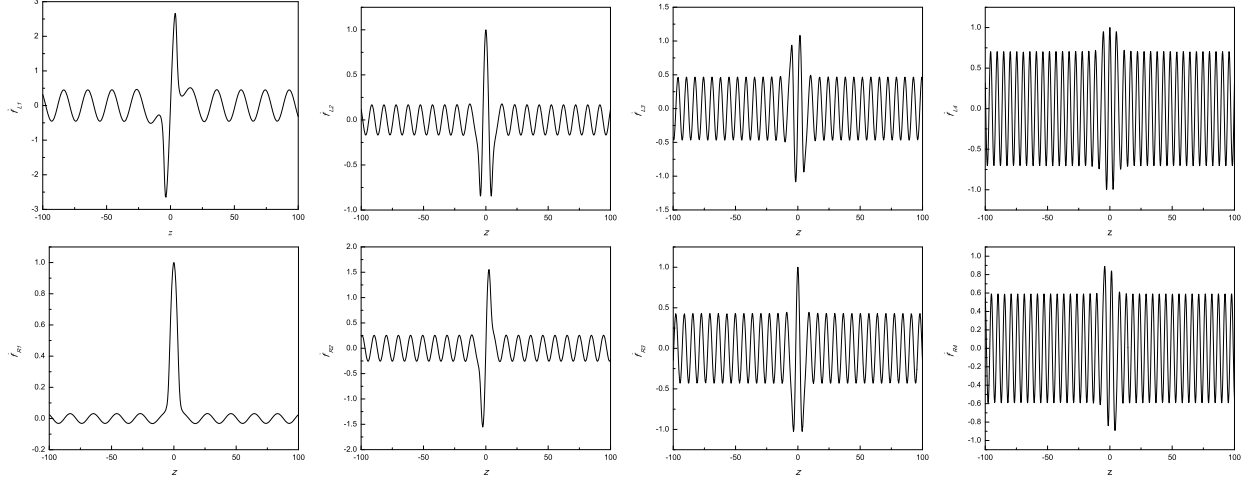


Figure 6: Plots of the left-handed and right-handed fermion resonances with  $\eta = 1$ ,  $\lambda = 1$  and  $a = 5$ .

not be normalized because they are oscillating when far away from brane along extra dimension. Therefore, the relative probability function is defined as [46]

$$P_{Ln,Rn}(m) = \frac{\int_{-z_b}^{z_b} dz |\tilde{f}_{Ln,Rn}(z)|^2}{\int_{-z_{max}}^{z_{max}} dz |\tilde{f}_{Ln,Rn}(z)|^2} \quad (58)$$

where  $2z_b$  is brane thickness approximately and  $z_{max} = 10z_b$  here.  $\tilde{f}_{Ln,Rn}(z)$  is solution of eq. (25) or (26) with two boundary conditions:

$$\tilde{f}_{Ln,Rn}(0) = 1, \quad \tilde{f}'_{Ln,Rn}(0) = 0, \quad (59)$$

for even parity, and

$$\tilde{f}_{Ln,Rn}(0) = 0, \quad \tilde{f}'_{Ln,Rn}(0) = 1 \quad (60)$$

for odd parity. If  $m^2 \gg V_{L,Rmax}$ ,  $\tilde{f}_{L,R}$  will be approximately a plane wave with  $P_{L,R}(m) = z_b/z_{max} = 0.1$ . For simplicity we concern  $\beta = 1$ . Figs. 4 and 5 show plots of the relative probability for different values of  $a$  and  $\eta$  for left-handed and right-handed fermions respectively. The masses of resonances were presented in Table 1. For simplicity the fermion resonant wavefunctions for  $\eta = 1$ ,  $\lambda = 1$  and  $a = 5.0$  were showed in Fig. 6.

From the figures, we can see that the spectra of massive KK modes of left handed and right handed fermions are almost the same which reveals that a Dirac fermion composed from left handed and right handed bound KK modes. Furthermore for fixed value of  $a$  the larger value of coupling parameter, the larger number of resonances. Also for fixed value of  $\eta$ , the larger value of  $a$  leads to larger number of peaks. The first peak is the most narrow and the resonances will become broader with increasing  $m$ . This means that first resonance has larger lifetime and the lifetime decline with increasing the mass of peak.

We can also see from figures that there are successively even and odd parity wavefunctions for left and right chiral modes with the same values of  $m^2$ . In other words, the zero mode is beginning of left

chirality with even parity spectrums, therefore the two first resonances (if exist) with the same  $m^2$  are odd parity left chiral mode and even parity right chiral mode. Next the two second resonances (if exist) with the same  $m^2$  are even parity left chiral mode and odd parity right chiral mode.

Left	$\eta = 0.5$	$\eta = 1.0$	$\eta = 2.0$
$a = 1.0$	absent	1.185465	1.719258
$a = 3.0$	0.453441	0.562819;0.994057	0.689911;1.232968;1.664278; 2.034932
$a = 5.0$	0.288833;0.567862	0.337121;0.646626; 0.924378;1.198549	0.377447;0.731511;1.058468;1.359395; 1.631559;1.884065;2.146727
Right	$\eta = 0.5$	$\eta = 1.0$	$\eta = 2.0$
$a = 1.0$	absent	1.110830	1.711422
$a = 3.0$	0.442279	0.562819;0.994057	0.689911;1.232968;1.662400;2.027853
$a = 5.0$	0.283917;0.561504	0.337121;0.645689; 0.921857;1.190822	0.377447;0.731480;1.058468;1.359395; 1.631559;1.882145;2.139101

Table 1: Masses of resonances for different values of  $\eta$  and  $a$ .

## 5 Conclusions

In this paper we have studied the issue of the localization of fermion field on the double wall brane. This brane includes two scalar field coupled minimally to brane. For observing whether the zero mode can be localized on this brane or not, we use a Yukawa coupling between fermion and background scalar fields. By investigating normalization condition, we can see that fermion couples with summation of kinks. we found that there is a relation between coupling constant and  $\lambda$  parameter. Also we found that the zero mode of right handed fermion can not be localized on the brane.

The massive mode resonances were investigated numerically. From the volcano shape of effective potential for left handed fermions, it results that the spectrum is continuous and there is no gap between zero mode and KK excitation modes. Also larger values of coupling constant and the distance of sub-branes support more resonances in the spectrum. In one spectrum, heavier resonances have broader peaks rather than lighter ones. This means the lighter fermions couple stronger to the brane rather than heavier KK modes. because of very narrow peak, We may not see light resonances. Fortunately because of supersymmetric feature of Schrödinger-like equation, we have successively even and odd parity for left or right chirality modes with the same mass in the spectrum. This helps us to search a small region when a resonance peak is not seen. Therefore numeric procedure becomes simple and fast. The lifetime of a resonance is proportional to inverse of peak width at half maximum. Hence generally, light resonances have longer lifetimes.

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